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Extra Sheet – Calculating Consumer Surplus, Producer Surplus, and Welfare

In this extra sheet, we will consider how to calculate consumer surplus, producer surplus, social cost and the deadweight loss. These will be important concepts and it is useful to understand them in a good level of detail. I will focus on the example given in the tutorials because it covers both the consequences of taxation as well as the negative externality itself!

For more information or another example see:

https://www.youtube.com/watch?v=1mjPIxy8-80&t=227s

The channel that produces these videos is really good and can help with worked example for most questions both this year and next year!

<u>Question:</u>

Market supply in a competitive industry is *p* = *Q*. Demand is *p* = 100 - *Q*. Production creates pollution with a social cost of $1 per unit of output. In response to environmentalists, the government creates a tax of $2 per unit. Is overall welfare improved or reduced by the tax?

We are given the following information:

Supply: $p = Q$

Demand: $p = 100 - Q$

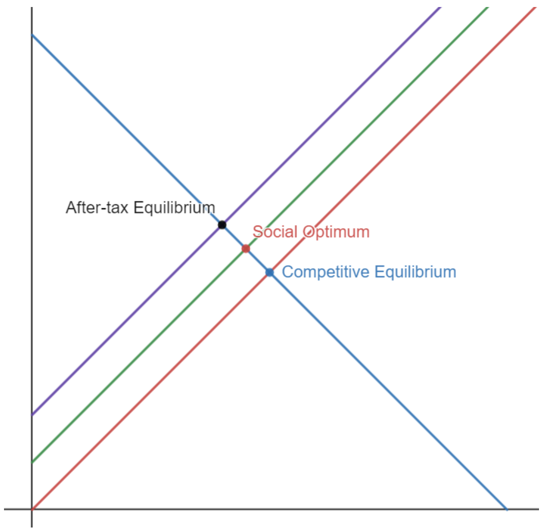
Externality: $E = \$1$ per unit

Tax: $\tau = \$2$ per unit

To start simply, let's first work out each of these equilibria and plot them on a diagram. This will make our welfare analysis a lot easier.

Perfect Competition	**Social Optimum**	**After-tax Equilibrium**
Need to use supply and demand function: $p = Q$ $p = 100 - Q$ In the equilibrium, supply is equal to demand. Hence: $Q = 100 - Q$ $2Q = 100$ $\boldsymbol{Q = 50, p = 50}$	At the social optimum, we must shift the supply curve upwards by $1 to take the externality into account: $p = Q + 1$ $p = 100 - Q$ In the equilibrium, supply is equal to demand. Hence: $Q + 1 = 100 - Q$ $2Q = 99$ $\boldsymbol{Q = 49.5, p = 50.5}$	At the after-tax equilibrium, we have simply shifted the supply curve up by $2 for the tax levied on producers: $p = Q + 2$ $p = 100 - Q$ In the equilibrium, supply is equal to demand. Hence: $Q + 2 = 100 - Q$ $2Q = 98$ $\boldsymbol{Q = 49, p = 51}$

The graph representing these equilibria is given below. It tells us that in the perfectly competitive outcome, we are over-producing, while in the after-tax equilibrium, we are under producing.



We now want to compare the welfare outcomes in each of these equilibria. Starting with perfect competition. I should say that even in the socially optimum, the negatively externality is not eliminated! The difference will that total welfare will be maximised there!

Competitive Equilibrium

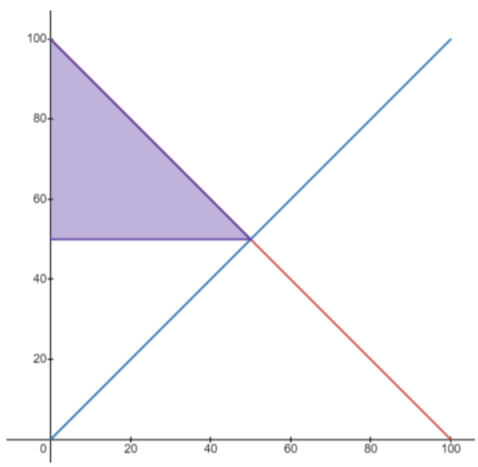
Consumer Surplus:

We want to find the area of this top triangle:

$$\boldsymbol{Q = 50, p = 50}$$

$$CS = \frac{1}{2} \times Q^* \times (100 - P^*)$$

$$CS = \frac{1}{2} \times 50 \times (100 - 50) = \mathbf{1250}$$



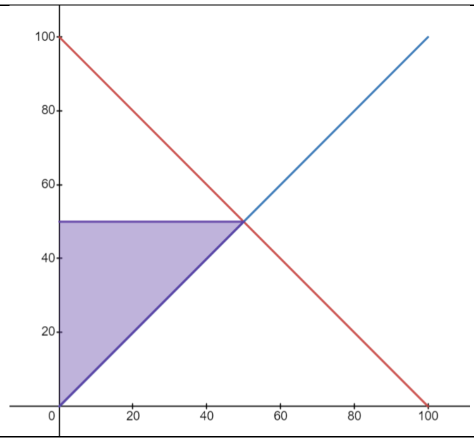
Producer Surplus:

We want to find the area of this top triangle:

$$\boldsymbol{Q = 50, p = 50}$$

$$PS = \frac{1}{2} \times Q^* \times (P^* - 0)$$

$$PS = \frac{1}{2} \times 50 \times (50 - 0) = \mathbf{1250}$$



Externality:

Recall that the externality is given in "per unit" terms. Hence its actual size depend on how much output is produced in equilibrium. In the perfectly competitive equilibrium, we have that the externality must be: $E = \$1 \times Q^* = \mathbf{50}$

Total Welfare:

We note that total welfare is then given by:

$$W = CS + PS - E = 1250 + 1250 - 50 = \mathbf{2450}$$

You may be wondering, where is the deadweight loss? This is a good question. We will see it when we compare the competitive equilibrium to the social optimal one!

Socially Optimal Equilibrium

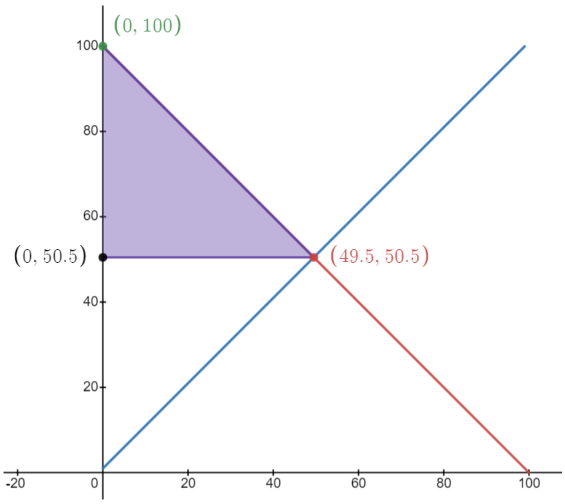
Consumer Surplus:

We want to find the area of this top triangle:

$$\boldsymbol{Q = 49.5, p = 50.5}$$

$$CS = \frac{1}{2} \times Q^* \times (100 - P^*)$$

$$CS = \frac{1}{2} \times 49.5 \times (100 - 50.5) = \mathbf{1225.125}$$



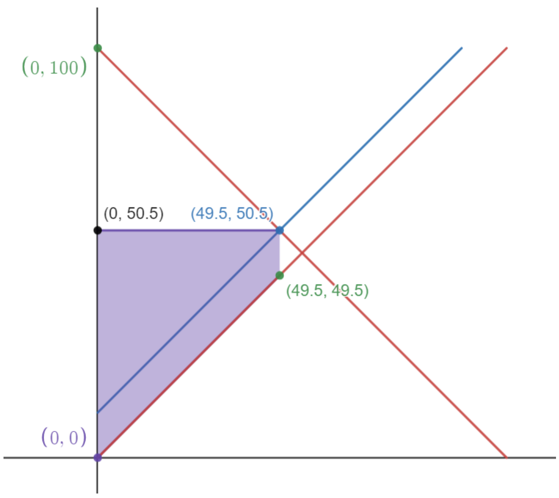
Producer Surplus:

Producer surplus is more complicated this time! Its note just a triangle anymore! You can see this from the diagram on the right. When we calculate producer surplus, we have to use the private marginal cost. This means that the producer surplus becomes a trapezium-like shape!

$$\boldsymbol{Q = 49.5, p = 50.5}$$

To find its area we use a geometric approach based on the image which yields:

$$PS = (50.5 - 49.5) \times 49.5) + \left(\frac{1}{2} \times 49.5 \times 49.5\right) = \mathbf{1274.625}$$



Externality:

Recall that the externality is given in "per unit" terms. Hence its actual size depend on how much output is produced in equilibrium. In the socially optimal equilibrium, we have that the externality must be: $E = \$1 \times Q^* = 49.5$

Total Welfare:

We note that total welfare is then given by:

$$W = CS + PS - E = 1225.125 + 1274.625 - 49.5 = \mathbf{2450.25}$$

Equivalently, we can find the total welfare just finding the area of the large triangle bounded by $p = Q + 1$ and $p = 100 - Q$. This gives us:

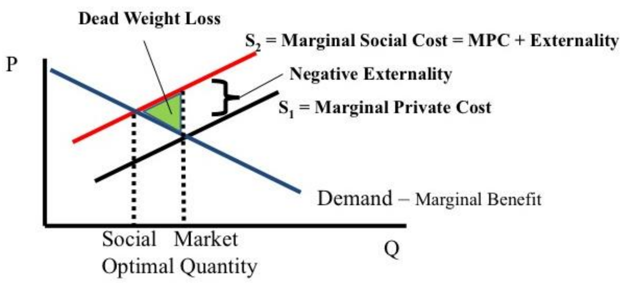
$$W = \frac{1}{2} \times (100 - 1) \times 49.5 = \mathbf{2450.25}$$

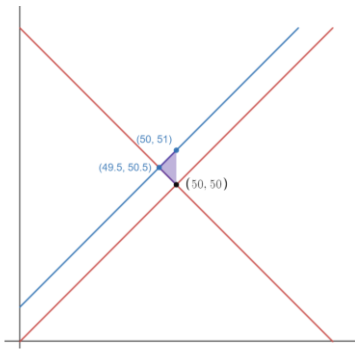
This reason this works is because it automatically takes the externality into account which in the long calculation above cancels with part of the producer surplus.

What about the deadweight loss? Technically speaking the deadweight loss is the difference between the societally optimal welfare compared to the scenario we are considering. In this case, the difference is:

$$DWL = 2450.25 - 2450 = 0.25$$

This is just the area of the normal triangle that we were looking at:





Considering this question mathematically, we have that:

$$DWL = \frac{1}{2} \times \$1 \times (Q_c^* - Q_{soc}^*) = \frac{1}{2} \times \$1 \times (50 - 49.5) = 0.25$$

Notice that this gives us the same answer as before! Indeed, this follows from the definition for the deadweight loss. Notice how this works for the other cases like monopoly vs perfect competition too! The triangle lost there will just be the total welfare in the perfectly competitive market minus the welfare under a monopoly. Now, we can use the same approach to analyse the effects of government intervention.

After-tax Equilibrium

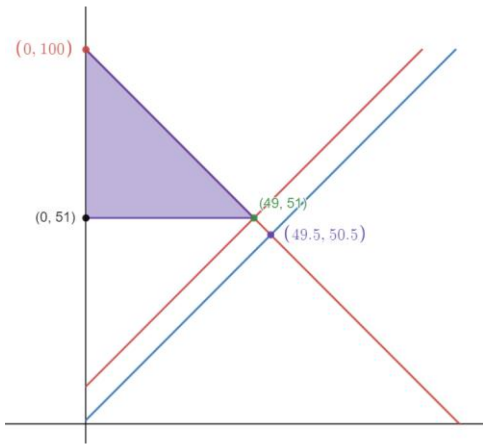
Consumer Surplus:

We want to find the area of this top triangle:

$$\boldsymbol{Q = 49, p = 51}$$

$$CS = \frac{1}{2} \times Q^* \times (100 - P^*)$$

$$CS = \frac{1}{2} \times 49.5 \times (100 - 50.5) = \mathbf{1225.125}$$



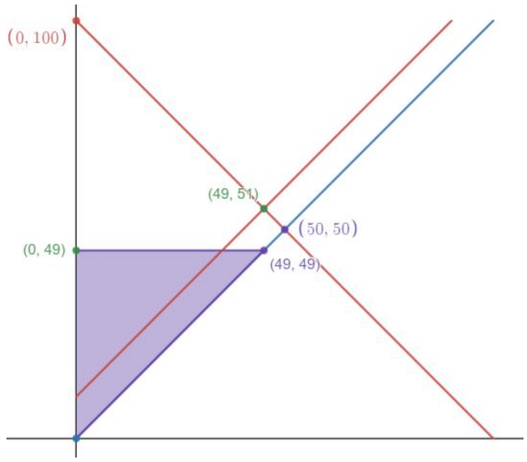
Producer Surplus:

Producer surplus is actually easier to understand here! It will look very similar to what happens when we have a monopoly!

$$\boldsymbol{Q = 49, p = 51}$$

To find its area we use a geometric approach based on the image which yields:

$$PS = \frac{1}{2} \times 49 \times 49 = \mathbf{1200.5}$$

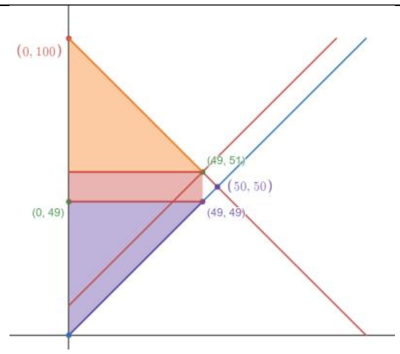


Government Tax Revenue:

We cut put some of the things we saw above together in one diagram. The orange triangle is consumer surplus, the purple triangle is producer surplus, and the red square is the government tax revenue:

$$G = \tau Q^* = 2 \times 49 = \$98$$

The little white triangle is the deadweight loss relative to the competitive equilibrium (ignoring the externality). The triangle would be similar but smaller relative to the social optimum.



Externality:

Recall that the externality is given in "per unit" terms. Hence its actual size depend on how much output is produced in equilibrium. In the socially optimal equilibrium, we have that the externality must be: $E = \$1 \times Q^* = 49$

Total Welfare:

We note that total welfare is then given by:

$$W = CS + PS + G - E = 1200.5 + 1200.5 + 98 - 49 = \mathbf{2450}$$

In this case, the difference between the welfare at the social optimum and after-tax equilibrium is:

$$DWL = 2450.25 - 2450 = 0.25$$

Notice that this is the same as the situation when we had no tax at all.