Externalities and Equilibrium

Preliminary Mathematical Note:

As an important disclaimer, this last topic is quite important and conceptually rich. As you will see below, there are not many mathematical exercises to try. This is mainly because this notion of general equilibrium can become difficult very quickly. For example, we could add firms into the mix too as well as intertemporal choices.

Suppose that we have an economy with two goods x and y. Consumers 1 and 2 each have positive endowments of both goods. We denote these as $\omega_1 = (x_1^e, y_1^e)$ and $\omega_2 = (x_2^e, y_2^e)$. Each consumer also has a utility function over these two goods: $U_1(x, y)$ and $U_2(x, y)$. We have seen that a Pareto efficient equilibrium can be found if:

$$MRS_1 = MRS_2$$

Moreover, in this equilibrium it must be the case that the equilibrium allocation does not exceed the aggregate endowment. That is that:

$$x_1 + x_2 = (x_1^e + x_2^e)$$

$$y_1 + y_2 = (y_1^e + y_2^e)$$

By solving this set of conditions simultaneously, we can find the *contract curve* for each individual. For example, suppose that the utility functions for each consumer are:

$$U_1(x, y) = x_1^{0.5} y_1^{0.5}$$

$$U_2(x, y) = x_2^{0.5} y_2^{0.5}$$

Suppose further that:

$$x_1 + x_2 = 50$$

$$y_1 + y_2 = 25$$

This means that between the two consumers, they have 50 units of good x and 25 units of good y. We now need to use our first rule, namely that the MRS of both consumers must be the same. Hence:

$$MRS_1 = MRS_2 \Longrightarrow \frac{y_1}{x_1} = \frac{y_2}{x_2}$$

Hence, we can write:

$$y_1 x_2 = x_1 y_2$$

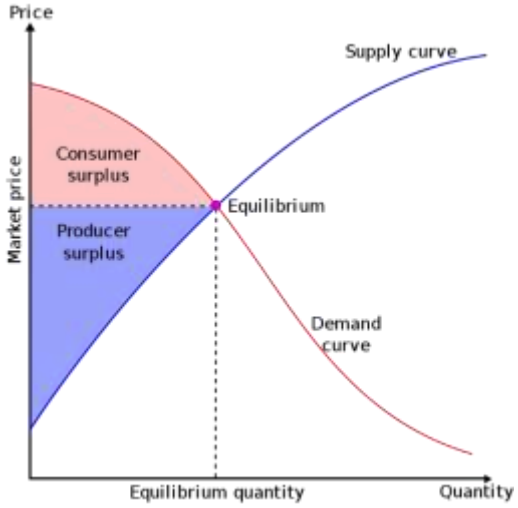
Now using the aggregate endowments, we can simplify this equation to:

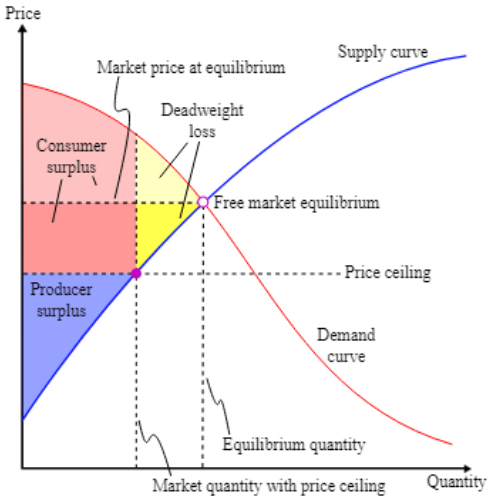
$$x_2 = 50 - x_1;\ y_2 = 25 - y_1$$

$$y_1(50 - x_1) = x_1(25 - y_1) \Rightarrow \boldsymbol{x_1 = 2y_1}$$

The same will hold for the other consumer. Hence, the contract curve will be $\boldsymbol{y = 0.5x}$, if plotted in the Edgeworth box.

Another aspect to all of this is the evaluation of welfare using notions of deadweight loss, consumer and producer surplus. The diagrams below should be familiar and illustrate each of these quantities. Our aim will be to quantify these mathematically, which can either be done using the area of a triangle or through integration.





Let's consider an example. Suppose that the demand curve is given by $p = 100 - 2Q$, while the supply curve is $p = 10 + Q$. The equilibrium is then $Q = 30, p = 40$. There are two ways of working out consumer and producer surplus:

Geometric	**Integration**
The idea is to merely work out the area of the relevant triangle using $A = \frac{1}{2}bh$. To find the base, we just need to the equilibrium output. To find the height, we need the difference between the y-intercept of the demand curve and the equilibrium price. Hence, $$CS = \frac{1}{2} \times 30 \times (100 - 40) = \mathbf{900}$$ The same idea holds for producer surplus. In this latter case, however, we would need to find the area of the bottom triable instead.	The idea here is that consumer surplus is equal to the difference between two areas. The first is the area underneath the demand curve between the y-axis and the equilibrium quantity. The second is the square bounded by the equilibrium point. Formally: $$CS = \int_0^{Q^*} D(Q)dQ - P^*Q^*$$ Hence, $$CS = \int_0^{30} (100 - 2Q)dQ - 1200$$ $$= [100Q - Q^2]_0^{30} - 1200$$ $$= 3000 - 900 - 1200 = \mathbf{900}$$

Both methods arrive at the same solution. We would use the same argument for finding the deadweight loss associated with a given market failure be that a negative externality or monopoly power.

Practice Problems

1. Work out the consumer surplus or producer surplus (depending on whether supply or demand are given) for the following:

a. $p = 15 - Q_d, p^e = 10$
b. $p = 5 + 2Q_s, Q^e = 4$

2. Consider an economy with two goods and two agents, Elisa and Bob. Elisa is initially endowed with $E = (5, 30)$ and Bob's endowment is $B = (20,20)$. Elisa and Ben have the same utility given by $U(x_1, x_2) = 2\ln x_1 + 2\ln x_2$.

    a. What is the marginal rate of substitution of each individual?
    b. Derive the contract curve for this economy.

3. Consider an economy with two goods and two agents, Elisa and Bob. Elisa is initially endowed with $E = (10, 20)$ and Bob's endowment is $B = (40,40)$. Elisa and Bob have utility functions given by $U_E(x, y) = 5x_E^{0.5}y_E^{0.5}$ and $U_B(x, y) = 2x_B^{0.1}y_B^{0.8}$.

    a. What is the marginal rate of substitution of each individual?
    b. Derive the contract curve for this economy.

4. Suppose that consumer A has a utility function of $U_A(x, y) = \min\{x_A, y_A\}$ while consumer B has the utility function $U_B(x, y) = x_B y_B$. Suppose that their respective endowments are $A = (1, 2)$ and $B = (1, 2)$.

    a. If a benevolent central planner reassigns the endowments such that now $A = (1, 1)$ and $B = (2, 2)$. Is this new allocation a Pareto improvement over the original one? Is it Pareto optimal, that is, can we find another allocation that can make at least one consumer better off and no one worse off)?
    b. Can a central planner achieve an allocation such that $A = (2, 2)$ and $B = (2, 2)$? If not, why not?
    c. (***Extension***) Suppose we made consumer A altruistic such that their new utility function is now $U_A(x, y) = \min\{4x_A, y_B\}$ and now depends on the consumption of good y by consumer B. What would be the Pareto optimal allocation now?

5. Work out the consumer and producer surplus for the following equilibria:

    a. $p = 18 - 0.5Q; p = \frac{1}{3}Q + 3$
    b. $P = 240 - 6Q; P = 120 + 4Q$

6. Now suppose that for both equilibria considered in the previous question, the government introduces a £2 per unit tax. Calculate the new consumer surplus, producer surplus as well as the deadweight loss to society from this tax.

7. Market supply in a competitive industry is $p = Q + 10$, while demand is $p = 50 - 2Q$. Suppose that that in production process imposes a social cost of $3 per unit of output. The government attempts to correct this problem by imposing a $2 tax on the producers. What effect does this policy have on consumer and producer surplus? What happens to the deadweight loss to society?

8. (***Challenge***) Suppose that consumer A has the utility function $U_A(x, y) = x_A^a y_A^b$, and consumer B has the utility function $U_B(x, y) = x_B^c y_B^d$. Suppose further that the aggregate endowment of each good is given by $x_A + x_B = X$ and $y_A + y_B = Y$ respectively. Derive the contract curve for this economy.

9. (***Challenge***) Suppose that the inverse demand curve for a monopoly market is $P = 30 - 2Q$, while the marginal cost of the monopolist is simply $MC = 12$.

    a. Calculate the output and price in the monopoly market.
    b. Calculate the equilibrium if this market was instead perfectly competitive.
    c. Find the consumer surplus for the cases in part (a) and part (b). Comment on how they compare to one another.