Factor Markets – SOLUTIONS

Practice Problems

1. For following firms operating in a competitive labour market, work out the number of workers the firms would hire.

   a. $w = 5, p = 10, Q = 2L^{0.5}$

$$5 = 10 \times L^{-0.5} \Rightarrow \boldsymbol{L = 4}$$

   b. $w = 10, MC = 5, Q = 30L^{\frac{1}{3}}$

$$10 = 5 \times 10L^{-\frac{2}{3}} \Rightarrow \boldsymbol{L \approx 11.18}$$

2. Work out the labour demand curve for the following monopoly firms.

   a. $p = Q^{-2}, Q = L^{0.5} + \frac{1}{L}$

$$MRP_L = MR \times MP_L = -\boldsymbol{Q^{-2}} \times \left(\frac{\boldsymbol{1}}{\boldsymbol{2\sqrt{L}}} - \frac{\boldsymbol{1}}{\boldsymbol{L^2}}\right)$$

You could leave it like that or simplify further by substituting in the production function:

$$MRP_L = -\left(\boldsymbol{L^{0.5}} + \frac{\boldsymbol{1}}{\boldsymbol{L}}\right)^{\boldsymbol{-2}} \times \left(\frac{\boldsymbol{1}}{\boldsymbol{2\sqrt{L}}} - \frac{\boldsymbol{1}}{\boldsymbol{L^2}}\right)$$

This could be made even simpler but hopefully you get the idea!

   b. $p = 100 - Q^2, Q = 2L^{0.5}$

$$MRP_L = MR \times MP_L = (\boldsymbol{100 - 3Q^2}) \times \frac{\boldsymbol{1}}{\boldsymbol{\sqrt{L}}} = \frac{\boldsymbol{100 - 12L}}{\boldsymbol{\sqrt{L}}}$$

3. Work out the equilibria for the following monopsony markets.

   a. $p = Q_s - 2; p = -2Q_d + 10$

$$ME = 2Q - 2$$
$$MV = -2Q + 10$$
$$ME = MV \Rightarrow 2Q - 2 = -2Q + 10$$
$$\boldsymbol{Q = 3, p = 1}$$

   b. $Q_s = 3p - 12;\ Q_d = -3p + 30$

$$ME = \frac{2Q}{3} + 4$$

$$MV = -\frac{Q}{3} + 10$$

$$ME = MV \Rightarrow \frac{2Q}{3} + 4 = -\frac{Q}{3} + 10$$

$$\boldsymbol{Q = 6, p = 6}$$

4. Consider a monopolist facing an inverse demand curve of $p = 150 - Q$. Suppose further that the firm uses the simple production function $Q = \sqrt{L}$ and the monopoly hires workers from a perfectly competitive labour market with an equilibrium wage rate of $w = 5$.

   a. What is the equilibrium quantity and price in the product market?

$$MC = \frac{w}{MP_L} = \frac{5}{\frac{1}{2\sqrt{L}}} = 10Q$$

$$MR = 150 - 2Q$$

$$MC = MR \Rightarrow 10 = 150 - 2Q$$

Applying quadratic formula:

$$\boldsymbol{Q = 12.5, p = 137.5}$$

   b. How many workers does the monopoly hire at the prevailing wage rate?

The equilibrium condition is simply:

$$w = MR \times MP_L \Rightarrow 5 = MR \times MP_L$$

$$MR = 150 - 2Q = 150 - 2\sqrt{L}$$

$$MP_L = \frac{1}{2\sqrt{L}}$$

Using the equation:

$$5 = \left(150 - 2\sqrt{L}\right)\frac{1}{2\sqrt{L}} \Rightarrow 6 = \frac{150}{2\sqrt{L}} \Rightarrow \boldsymbol{L = 156.25 = 12.5^2}$$

   c. Now work out the equilibrium output, price, and employment if the product market was perfectly competitive.

A competitive market is defined using $p = MC$. Hence, we have that:

$$p = 10Q$$

Thus,

$$10Q = 150 - Q \Rightarrow \boldsymbol{Q = \frac{150}{11} \approx 13.63, p = 136.36}$$

There is a shortcut to finding employment: $Q = \sqrt{L} \Rightarrow \boldsymbol{L = Q^2 \approx 185.95}$. Note that using the normal route using the marginal product of labour would yield the same solution.

d. Comment on the differences between these two equilibria and explain why they arise.

You can see that employment and output are higher in the competitive equilibrium. This should not be unexpected as a monopoly restricts output to earn a supernormal profit. Afterall, it faces a downward sloping marginal revenue curve unlike the competitive market firms.

5. The employment of PGTAs in many universities can be characterized as a monopsony. Suppose the demand for PGTAs is given by $w = 30{,}000 - 100L$. The supply of PGTAs is given by $w = 1500 + 50L$.

   a. If universities take advantage of their monopsonist position, how many PGTAs will be hired? What will be the equilibrium wage?

$$ME = 1500 + 100L$$
$$MV = 30{,}000 - 100L$$
$$ME = MV \Longrightarrow 1500 + 100L = 30{,}000 - 100L \Longrightarrow \boldsymbol{L = 142.5}$$
$$w = 1500 + 50 \times 142.5 = \mathbf{8625}$$

   b. Find the equilibrium if the market was perfectly competitive.

   In the perfectly competitive case, supply equals demand:

$$1500 + 50L = 30{,}000 - 100L \Rightarrow \boldsymbol{L = 190, w = 11000}$$

   c. Suppose that universities face an infinite supply of PGTAs at the annual wage level of $w = 10{,}000$. How many will be hired at this wage?

$$ME = 10000$$
$$MV = 30{,}000 - 100L$$
$$ME = MV \Longrightarrow 10000 = 30{,}000 - 100L \Longrightarrow \boldsymbol{L = 200}$$

6. Consider some industry with a perfectly competitive labour market. Suppose that in this case, the labour supply curve is upward sloping such that $w = L + 2$. Suppose further than the marginal product of labour is given by: $MP_L = 10 - 0.5L$, and that the product market price is $p = 2$.

   a. Find the equilibrium wage and the level of employment. Is there unemployment?

$$L + 2 = 2(10 - 0.5L) \Rightarrow L + 2 = 20 - L \Longrightarrow \boldsymbol{L = 9, w = 11}$$

   There is no unemployment.

   b. Suppose that the demand in the product market increased, comment on what the effect that would have on the labour market.

An increase in demand would increase the equilibrium price in the product market. This would increase demand for labour, thus yielding an equilibrium with a higher level of employment.

c. Now suppose that there is a monopsony in the labour market. What is the equilibrium wage and employment.

The demand for labour stays the same. However, we now need to consider marginal expenditure and set it equal to demand:

$$ME = 2L + 2$$
$$MV = 2(10 - 0.5L)$$
$$ME = MV \Longrightarrow 2L + 2 = 2(10 - 0.5L) \Longrightarrow \boldsymbol{L = 6, w = 8}$$

d. (***Challenge***) The government being dissatisfied with the monopsony decides to strike a detail. It proposes to give workers a monetary reward to join the labour market. What should the per worker reward be so that the new equilibrium is the same as the one you found in part (a).

The following equation holds:

$$ME = 2L + 2 + \tau$$
$$MV = 2(10 - 0.5L)$$

Here τ denotes a transfer. The point being that we need to find its value such that the equilibrium employment is 9. Thus:

$$2(9) + 2 + \tau = 2\big(10 - 0.5(9)\big) \Longrightarrow \boldsymbol{\tau = -9}$$

This looks somewhat unintuitive. However, this is because we are looking at the inverse supply curve. In reality, we have shifted the labour supply curve such that workers are willing to work more for any given wage rate.

7. (***Challenge***) Suppose that a monopsony faces a supply curve of the form $p = p(Q)$, where $p(Q)$ is some function of quantity supplied.

a. By applying the product rule of differentiation, show that marginal expenditure can be written as:

$$ME = AE + \frac{dp}{dQ}Q$$

We recall that total expenditure can be written as:

$$TE = Qp = Q \times p(Q)$$

To find marginal expenditure we find the derivative with respect to quantity:

$$ME = p(Q) + p'(Q)Q = p + \frac{dp}{dQ}Q$$

The last thing to note is that average expenditure is just $\frac{TE}{Q} = \frac{Qp}{Q} = p$. Hence, by substitution:

$$\boldsymbol{ME = AE + \frac{dp}{dQ}Q}$$

b. Now show that this equation can be rewritten as:

$$ME = p\left(1 + \frac{1}{\varepsilon}\right)$$

We recall that average expenditure is just price. Hence:

$$ME = p + \frac{dp}{dQ}Q$$

$$ME = p\left(1 + \frac{dp}{dQ}\frac{Q}{p}\right)$$

Using the definition of elasticity, we can see that $\frac{1}{\varepsilon} = \frac{dp}{dQ}\frac{Q}{p}$. Hence the equation simplifies to the required result:

$$\boldsymbol{ME = p\left(1 + \frac{1}{\varepsilon}\right)}$$

c. Comment on whether the elasticity in the above equation is positive or negative. Why?

The elasticity is positive because it represents the elasticity of supply. In both the labour and product market, firms respond positively to an increase in price.