Monopoly and Oligopoly

Preliminary Mathematical Note:

Given the wide range of material needed here, I will cover some of the most important results. For a general inverse demand function, $p = a - bQ$, the marginal revenue can be found as follows:

$$TR = pQ = (a - bQ)Q \Rightarrow MR = \frac{dTR}{dQ} = a - 2bQ$$

However, marginal revenue also has a more abstract form. Consider again the equation $TR = pQ$. We know that price is a function f total output. Hence, applying product rule we have that:

$$MR = \frac{dTR}{dQ} = p + \frac{dp}{dQ}Q$$

Recall that elasticity of demand is defined as: $\varepsilon = \frac{dQ}{dp}\frac{p}{Q}$. This implies that $\frac{p}{\varepsilon} = \frac{dp}{dQ}Q$. By substitution, we have that:

$$MR = p + \frac{p}{\varepsilon} = p\left(1 + \frac{1}{\varepsilon}\right)$$

This links together elasticity of demand with marginal revenue. This elasticity will determine how changes to marginal cost will affect the price (in a monopoly). For example, we may wish to analyse how changes to tax affect prices. We can work this out as follows. To profit maximise, firms will set marginal revenue equal to marginal cost, hence, from above:

$$MC = p\left(1 + \frac{1}{\varepsilon}\right) \Longrightarrow p = \frac{MC}{\left(1 + \frac{1}{\varepsilon}\right)}$$

Taxes increase marginal costs, and these will affect price. How much is passed through will depend on the elasticity of demand. We can calculate the incidence of tax on consumers using the following formula:

$$i = \frac{\Delta p}{\Delta t}$$

Note that a monopoly will usually produce less than a perfectly competitive firm would. This will often lead to a deadweight loss to society which arises when the price exceeds the marginal cost. This DWL can be found by using the equation $\frac{1}{2}(P - MC)\left(Q_{pc} - Q_m\right)$: the height of the triangle is the difference between price and marginal cost, and its base is the difference in output between perfect competition and a monopoly.

In addition to PC and monopolies, there are other market structures such as oligopolies. In these there are several large firms whose joint production influences the market price. To understand equilibria in these cases, we must use tools from game theory. We consider three models: Cournot, Bertrand (differentiated) and Stackelberg, and will solve each for their Nash equilibrium.

I will not that there are formulae that one can use to speed up the process, but they are not always applicable. Indeed in the cases worked out below, they will not work as the problems are asymmetric.

Cournot – Worked Example

Suppose that there are two firms with marginal costs of 2 and 4 respectively. The market inverse demand curve is $p = 100 - q_1 - q_2$. Find the Nash Cournot equilibrium.

Solution

To solve this problem, we follow three steps: (1) Profit Maximise, (2) Find reaction functions and (3) Solve reaction functions simultaneously.

<table>
<tr><th>Firm 1</th><th>Firm 2</th></tr>
<tr><td>Step 1: Profit Maximisation
$TR = pq_1 = 100q_1 - q_1^2 - q_2q_1$
$MR = 100 - 2q_1 - q_2$
$MC = 2$
$MR = MC \Rightarrow \mathbf{100 - 2q_1 - q_2 = 2}$</td><td>Step 1: Profit Maximisation
$TR = pq_2 = 100q_2 - q_2^2 - q_1q_2$
$MR = 100 - 2q_2 - q_1$
$MC = 4$
$MR = MC \Rightarrow \mathbf{100 - 2q_2 - q_1 = 4}$</td></tr>
<tr><td>Step 2: Reaction Functions
$100 - 2q_1 - q_2 = 2$
$-2q_1 = -98 + q_2$
$\boldsymbol{q_1^* = \frac{98 - q_2}{2}}$</td><td>Step 2: Reaction Functions
$100 - 2q_2 - q_1 = 4$
$-2q_2 = -96 + q_1$
$\boldsymbol{q_2^* = \frac{96 - q_1}{2}}$</td></tr>
<tr><td colspan="2">Step 3: Solve Simultaneously
Substitute the reaction function for firm 2 into firm 1’s reaction function and solve for NE:
$$q_1^* = \frac{98 - \frac{96 - q_1}{2}}{2} = 49 - 24 + \frac{1}{4}q_1^*$$
$$\frac{3}{4}q_1^* = 25$$
$$\boldsymbol{q_1^* = \frac{100}{3} \approx 33.33}$$
By substitution into firm 2’s reaction function
$$\boldsymbol{q_2^*} = \frac{96 - \frac{100}{3}}{2} = \boldsymbol{\frac{94}{3} \approx 31.33}$$
The equilibrium price can again be found by substitution:
$$p^* = 100 - \frac{100}{3} - \frac{94}{3} = \boldsymbol{\frac{106}{3} \approx 35.33}$$
Notice that the firm with the lower marginal cost produces more in Nash equilibrium! This makes sense because it has a higher productivity.</td></tr>
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Bertrand – Worked Example

Suppose that there are two firms with marginal costs of 2 and 4 respectively. The demand curves for each of these firms is $q_1 = 100 - p_1 + p_2$ and $q_2 = 120 - 2p_2 + p_1$. Find the Nash Cournot equilibrium.

Solution

To solve this problem, we follow three steps: (1) Profit Maximise, (2) Find reaction functions and (3) Solve reaction functions simultaneously. Same as with the Cournot case, just with price competition.

Firm 1	**Firm 2**
Step 1: Profit Maximisation $\pi_1 = (p_1 - 2)q_1 = (p_1 - 2)(100 - p_1 + p_2)$ $\frac{d\pi}{dp_1} = (100 - p_1 + p_2) - (p_1 - 2) = 0$	**Step 1: Profit Maximisation** $\pi_2 = (p_2 - 4)q_2 = (p_2 - 4)(120 - 2p_2 + p_1)$ $\frac{d\pi}{dp_2} = (120 - 2p_2 + p_1) - 2(p_2 - 4) = 0$
Step 2: Reaction Functions $(100 - p_1 + p_2) - (p_1 - 2) = 0$ $2p_1 = 102 + p_2$ $\boldsymbol{p_1^* = \frac{102 + p_2}{2}}$	**Step 2: Reaction Functions** $(120 - 2p_2 + p_1) - 2(p_2 - 4) = 0$ $4p_2 = 128 + p_1$ $\boldsymbol{p_2^* = \frac{128 + p_1}{4}}$

Step 3: Solve Simultaneously

Substitute the reaction function for firm 2 into firm 1's reaction function and solve for NE:

$$p_1^* = \frac{102 + \frac{128 + p_1}{4}}{2} = 51 + 16 + \frac{1}{8}p_1^*$$

$$\frac{9}{8}p_1^* = 67$$

$$\boldsymbol{p_1^* = \frac{536}{9} \approx 59.56}$$

By substitution into firm 2's reaction function

$$\boldsymbol{p_2^* = \frac{128 + \frac{536}{9}}{4} = \frac{422}{9} \approx 46.89}$$

The equilibrium quantities can again be found by substitution into the two respective demand functions (if needed).

Stackelberg – Worked Example

Suppose that there are two firms each with marginal costs of 2. The market inverse demand curve is $p = 100 - q_1 - q_2$. Find the Stackelberg equilibrium in which firm 1 is the leader and firm 2 is the follower.

Solution

In this case decision are made sequentially. Hence, we use backward induction to find the equilibrium. We first consider the profit maximisation problem for firm 2, then we will see how firm 1 devises a strategy in light of this information.

To solve this problem, we follow four steps: (1) Profit maximisation for follower firm, (2) Find reaction function for follower firm, (3) Substitute reaction function into leader firm's profit maximisation problem and, (4) Profit maximisation for leader firm.

Firm 2 (follower)
Step 1: Profit Maximisation
$TR = pq_1 = 100q_2 - q_2^2 - q_1 q_2$
$MR = 100 - 2q_2 - q_1$
$MC = 2$
$MR = MC \Rightarrow \mathbf{100 - 2q_2 - q_1 = 2}$

Step 2: Reaction Functions

$$100 - 2q_2 - q_1 = 4$$
$$-2q_2 = -98 + q_1$$
$$\boldsymbol{q_2^* = \frac{98 - q_1}{2}}$$

Step 3: Firm 1 (leader) profit maximisation
We substitute the reaction function for the follower firm into the inverse demand function. Hence, total revenue for firm 1 is now given by:

$$TR = pq_1 = 100q_1 - q_1^2 - q_1 \frac{98 - q_1}{2}$$

The reason we do this is because firm 1 now expects firm 2 to behave rationally and respond in this way. This happens because of the sequential nature of the model. We now profit maximise as usual:

$MR = 100 - 2q_1 - \left(\frac{98 - q_1}{2} - 0.5q_1\right)$
$MC = 2$
$MR = MC \Rightarrow 100 - 2q_1 - \left(\frac{98 - q_1}{2} - 0.5q_1\right) = 2$
$100 - 2q_1 - 49 + q_1 = 2$
$\boldsymbol{q_1^* = 49}$

By substitution into firm 2's reaction function

$$\boldsymbol{q_2^*} = \frac{98 - 49}{2} = \mathbf{24.5}$$

The equilibrium price can again be found by substitution:

$$p^* = 100 - 49 - 24.5 = \mathbf{26.5}$$

Notice that the firm which goes first has a large advantage even though both firms have the same marginal cost. This is called the ***first mover advantage.***

Note finally that we have been assuming that in the Bertrand case that the firms have differentiated products. In the undifferentiated case, price competition would drive down prices until they are equal to the marginal costs of both firms. As a final note, it may be possible for the firms to collude. In such a case, they would maximise their joint profits by collectively acting like a monopolist.

Practice Problems

1. Find the marginal revenue for a monopolist based on the following information:

a. $p = 250 - Q; MC = 10$
b. $E_d = -2; p = 25$

2. Consider the following monopoly markets each with a total cost function of $C(q) = Q^2 + Q$. Derive the equilibrium quantity and price for each of these.
   a. $p = 250 - 5Q$
   b. $10p = 500 - 25Q$
   c. $14Q = 490 - 7p$
   d. $p = 100 + Q - Q^2$
   e. $p = 300 - 5Q - Q^2$

3. Suppose that the elasticity of demand is constant at $E_d = -2$. Using the appropriate general equation for marginal revenue for a monopoly, explain what happens to price if marginal cost increases by τ? How does this depend on the elasticity of demand?

4. Draw a diagram for a typical monopolist with the demand curve (average revenue curve), the MR curve, the MC curve, and the AC curve. Shade and label the consumer and producer surplus, as well as the deadweight loss to society. Label and compare the quantity that would be produced if this were a perfectly competitive market.

5. Consider the following inverse demand function: $p = 500 - 2Q$. For this question assume that all firms have a marginal cost of \$2 per unit.
   a. Compare the incidence of a \$5 specific tax on consumers between a monopoly and a perfectly competitive firm.
   b. In each of four cases above, how much tax does the government collect?

6. Using the inverse demand function $p = 245 - 5Q$, and total cost function of $TC = 2Q^2 + 5$.
   a. Calculate the equilibrium for a monopoly and a perfectly competitive firm.
   b. What is the deadweight loss associated with monopoly production?

7. Consider the following market inverse demand curve: $p = 100 - 2Q$. Assume that all firms in this question, unless stated otherwise, have a total cost function of $TC = 4Q$.
   a. Solve for the monopoly equilibrium output and price.
   b. By assuming that $Q = q_1 + q_2$, find the Cournot Nash equilibrium.
   c. Recall the reaction functions that you found in part (b), are they increasing or decreasing in the quantity produced by the other firm. Explain the intuition behind this.
   d. Assume instead that quantities are chosen sequentially, find the Nash Stackelberg equilibrium when firm 1 is the leader and firm 2 is the follower.
   e. What difference is difference in terms of market price and profits between these three market structures?
   f. Suppose now that firm 1 has a marginal cost of \$8 per unit, while firm 2 has a marginal cost of \$4 per unit.
      i. Find the Cournot Nash equilibrium with these marginal costs.
      ii. What is the new Stackelberg equilibrium?
      iii. Comment on why these equilibria differ from those you found in earlier parts of the question.

8. Suppose that we have a market with two firms selling a differentiated product. Their respective demand curves are: $q_1 = 36 - 3p_1 + p_2$ and $q_2 = 24 + p_1 - 3p_2$.. The marginal cost of both firms is assumed to be $2.
   a. Find the Nash Bertrand equilibrium in prices and quantities.
   b. Recall the reaction functions that you found in part (a), are they increasing or decreasing in the price chosen by the other firm. Explain the intuition behind this.
   c. Calculate the market share of each firm.
   d. What happens to the market share of each firm when firm 2's marginal cost falls to $1 per unit? What happens to its Lerner index of market power before and after this change?
   e. (***Challenge***) Suppose now that prices are chosen sequentially such that firm 1 is the leader and firm 2 is the follower. What is the equilibrium now? Which firm has a greater market share?

9. (***Challenge***) Suppose that we observe the following inverse demand function $p = 36 - 3Q$ such that $Q = q_1 + q_2 + q_3$. Assume that all firms have no costs. Solve for the 3-firm Nash Stackelberg equilibrium where firm 1 chooses its output first, then firm 2 and lastly firm 3. Which firm produces most output? Why?

10. [1](***Challenge***) A differentiated Bertrand duopoly faces the demand curves given by:

$$q_1 = 1 - p_1 + \beta p_2 \text{ and } q_2 = 1 - p_2 + \beta p_1$$

Where q_i denotes the quantity demanded of each firm, p_i denotes the chosen price and β is a demand parameter. Assume that the marginal cost of production is zero.

   a. Suppose the firms play only once and set price simultaneously. Derive the best replies for each firm (reaction functions) and determine the Nash equilibrium. How does the equilibrium depend on β?
   b. Determine the most profitable collusive prices that maximises joint profits of both firms.

[1] Question taken from the UCLA Economics, Spring 2005, Second Year PhD Qualifying examination for Industrial Organisation. Good luck, O.B!