Competitive Firms and Markets

Preliminary Mathematical Note:

There are several important details to remember for competitive markets and how they work. The one that was covered is the concept of elasticity. This measures the responsiveness of a variable to a change in another. Its utility comes from the fact that it has not units and is expressed in percentage terms. Mathematically:

$$\varepsilon = \frac{dQ}{dP}\frac{P}{Q}$$

Here we have a demand/supply elasticity. This tells us by how many percentage points do demand/supply change when prices increase by 1%. Elasticities tell us a lot about the demand conditions of a market. For example, in competitive markets the residual demand curve for a firm is usually very elastic!

When operating in a market, firms make two decision: (1) How much to produce? (2) Should the firm operate? Output decisions are often stated in terms of profit maximisation. In most cases, profit maximisation occurs at the output level where marginal revenue is equal to marginal cost, or:

$$\max_q\{\pi\} \Leftrightarrow MC(q) = MR$$

In a perfectly competitive market where individual firms have minimal market power, marginal revenue is equal to price, hence we have the famous condition that $p = MC(q)$.

However, even if a firm is profit maximizing, it still may need to shut down, particularly if it is making a loss. In both the short and long run, a firm shuts down if its revenue is smaller that is avoidable costs. In other words, if it could reduce its losses by ceasing to operate. In the short-run, this is equivalent to saying that revenue is less than variable cost (or that price is less that minimum average variable cost). In the long run, this means that revenue is less than cost, and any loss is made (or price is less that minimum of average cost).

Lastly, we state that in the long run firms enter the market as long as profits are positive, then down to zero. Hence, in the long run firms operate at the lowest point of their average cost curves (assuming all firms are identical).

Practice Problems:

1. A consumer has a demand function for lemons of $q = 10 - 5p$. Find the elasticity of demand for lemons of this consumer, and comment on how it changes from $p = 0$ to $p = 2$.

2. Suppose that there are 5 identical firms in a competitive market such that the elasticity of demand is given by -0.5. Suppose further that the elasticity of supply is 4.4. Calculate the residual elasticity of demand for an individual firm.

3. Consider a competitive market of 15 firms, where each firm has a supply function of $q_i = \frac{2}{5}p$, and where the elasticity of demand is -1.5.

    a. Find the market supply function and derive the elasticity of supply for $n - 1$ firms.

b. Use these values to derive the residual elasticity of demand for an individual firm.

4. Prove that if a firm is maximising profits, then it produces at the output where its marginal revenue is equal to its marginal cost.

5. For each of the following cases determine whether the firm will close down or continue operating. Assume in each case that firms are in the short run:

   a. $p = 5; q = 20;\ VC = 4q;\ FC = 150$
   b. $TR = 125;\ VC = 126;\ FC = 1$
   c. $MR = p;\ TC = q^3; p = 3$
   d. $MR = p;\ TC = q^3 + 60; p = 27$

6. Consider the following situations and find the profit maximising level of output:
   a. $MC = q^2 - 2q + 1; MR = q - 1$
   b. $\pi(q) = -q^2 + 4q + 10$

7. A firm operating in a competitive market has two plants in which it can produce. The cost functions of each plant are: $C(q) = q^3 - 4q^2 + 2q$ and $C(q) = q^3 - 2q^2 + q$:
   a. Find the supply functions of each plant.
   b. Derive the profit function of the entire firm.

8. A competitive market operates in the short run with 4 firms. The market demand curve is given by: $Q = \frac{I+20p}{p}$, where I is aggregate consumer income. Suppose that the cost function of each firm is $C(q) = 5 + q^2$. Suppose that $I = 62.5$.
   a. Find the individual and market supply curve.
   b. What will be the equilibrium price?
   c. How much profit will each firm make?
   d. How do these change when consumer income falls to $I = 10.5$?

9. Suppose now that there is a competitive market operating in the long run with the demand function $Q = \frac{100+20p}{p}$. As in the previous question each firm that either operates in the market is looking to enter has the cost function $C(q) = 5 + q^2$. Use what you know about competitive markets to work out the equilibrium number of firms.

10. A firm operates with the production function $q = K^{0.5}L^{0.5}$. Assuming that inputs prices are $w = r = 1$.
    a. Finds its expansion path and cost function.
    b. From now on, suppose there are 19 other firms just like this one operating in a competitive market with demand function $Q = 100 - P$. Find the individual supply curve for each firm.

11. ***(Challenge)*** By using the residual demand curve, prove that the individual elasticity of demand for a firm is given by:

$$\varepsilon_i = n\varepsilon - (n-1)\eta_0$$

12. ***(Challenge)*** Suppose that we are operating in the short run in a competitive market. Suppose all firms are identical and have a Cobb-Doughlas production function of the form $q_i = K^{\alpha}L^{\beta}$. Suppose further that input prices are $w = r = 1$.

    a. Use the production function and given input prices to derive the expansion path of each firm.
    b. Use the expansion path to derive the cost function for each firm.
    c. State the profit maximisation problem.
    d. Use the fact that the firms are operating in a competitive market to derive the individual supply curve of each firm.
    e. Use the individual supply curve, derive the profit function of each firm.