Supply and Demand– SOLUTIONS

1. For each of the following demand or supply functions, state what will happen to quantity demanded/supplied if price increase from $p = 3$ to $p = 5$. Moreover, state what will happen to quantity demanded/supplied if price increase by £x.

   a. $2Q_d = 50 - 2p \Rightarrow Q_d^1 - Q_d^0 = -2; \frac{\Delta Q}{\Delta p} = 1$, hence an increase in £$x$ will reduce demand by x units.
   b. $3Q_s = 3 + 9p \Rightarrow Q_s^1 - Q_s^0 = 6; \frac{\Delta Q}{\Delta p} = 3$, hence an increase in £$x$ will increase supply by $3x$ units.

2. Given the quantity demanded and quantity supplied functions, find the equilibrium price and quantity:

   a. $Q_s = -5 + 3p$ and $Q_d = 10 - 2p$

$$Q_s = Q_d \Rightarrow -5 + 3p = 10 - 2p \Rightarrow 5p = 15 \Rightarrow \boldsymbol{p = 3}, \boldsymbol{Q = 4}$$

   b. $Q_s = -20 + 3p$ and $Q_d = 220 - 5p$

$$Q_s = Q_d \Rightarrow -20 + 3p = 220 - 5p \Rightarrow 8p = 240 \Rightarrow \boldsymbol{p = 30}, \boldsymbol{Q = 70}$$

   c. $Q_s = -45 + 8p$ and $Q_d = 125 - 2p$

$$Q_s = Q_d \Rightarrow -45 + 8p = 125 - 2p \Rightarrow 10p = 160 \Rightarrow \boldsymbol{p = 17}, \boldsymbol{Q = 93}$$

   d. $Q_s = -32 + 7p$ and $Q_d = 128 - 9p$

$$Q_s = Q_d \Rightarrow -32 + 7p = 128 - 9p \Rightarrow 16p = 160 \Rightarrow \boldsymbol{p = 10}, \boldsymbol{Q = 38}$$

   e. $Q_s = -27 + 13p$ and $Q_d = 24 - 4p$

$$Q_s = Q_d \Rightarrow -27 + 13p = 24 - 4p \Rightarrow 17p = 51 \Rightarrow \boldsymbol{p = 3}, \boldsymbol{Q = 12}$$

3. Supply and demand functions may also be quadratic. Find the equilibrium price and quantity when $p + Q_d^2 + 3Q_d = 20$ and $p - 3Q_s^2 + 10Q_s = 5$

$$p = 5 + 3Q_s^2 - 10Q_s$$

By substitution and recalling that $Q_s = Q_d$ in equilibrium:

$$4Q^2 - 7Q - 15 = 0$$

Factorising:

$$4Q^2 - 7Q - 15 = (4Q + 5)(Q - 3) = 0$$

Hence:

$$Q = -\frac{5}{4}; Q = 3$$

The first option is nonsense. Hence, $Q = 3$. By substitution, we find the equilibrium price:

$$p = 5 + 3(9) - 10(3) = 5 + 27 - 30 = 2$$

Hence, the equilibrium is given by: $(\boldsymbol{p = 2, Q = 3})$

4. Suppose that the demand and supply functions are linear. Suppose that when $p = 4$, quantity demanded is $Q_d = 31$, and when $p = 8$, quantity demanded is $Q_d = 11$. Suppose further that when $p = 8$, quantity supplied is $Q_s = 3$, and when $p = 12$, quantity supplied is $Q_s = 15$.
    a. Find the equation for quantity demanded.

$$m = \frac{y_1 - y_0}{x_1 - x_0} = \frac{31 - 11}{4 - 8} = -\frac{20}{4} = -5$$
$$y = -5x + c$$
$$c = y + 5x$$
$$c = 31 + 5(4) = 51$$
$$Q_d = -5p + 51$$

    b. Find the equation for quantity supplied.

$$m = \frac{y_1 - y_0}{x_1 - x_0} = \frac{3 - 15}{8 - 12} = \frac{12}{4} = 3$$
$$y = 3x + c$$
$$c = y - 3x$$
$$c = 3 - 3(8) = -21$$
$$Q_s = 3p - 21$$

    c. Find the equilibrium price and quantity.

In equilibrium, $Q_s = Q_d$. Hence:

$$-5p + 51 = 3p - 21 \Rightarrow 8p = 72 \Rightarrow \boldsymbol{p = 9; Q = 6}$$

5. Suppose that $Q_s = -5 + 2p$ and $Q_d = 10 - p$. Find the equilibrium price and quantity. Suppose now that the government imposes a price ceiling of £2 below the equilibrium. What is the quantity demanded and quantity supplied at this new price. What is the size of the shortage/surplus? Suppose instead that the government imposed a minimum price that is £1 above the original equilibrium. What is quantity demanded and quantity supplied? How large is the shortage/surplus?

$$Q_s = Q_d \Rightarrow -5 + 2p = 10 - p \Rightarrow \boldsymbol{p = 5, Q = 5}$$

At the price ceiling: $Q_s = 1 < Q_d = 7$. Hence the size of the shortage is 6 units. At the price floor: $Q_s = 7 < Q_d = 4$. Hence the size of the surplus is 3 units.

6. For the following demand and supply equations (taken from Q2 above) work out the effect of a specific tax of £3 per unit levied on producers:
   a. $Q_s = -5 + 3p$ and $Q_d = 10 - 2p$

      Supply curve shifts up by 3 units. New inverse supply curve is:

$$p = \frac{Q_s}{3} + \frac{14}{3} \Rightarrow Q_s = 3p - 14$$

      Can also be worked by substituting $p - 3$ into supply curve:

$$Q_s = -5 + 3(p - 3) = 3p - 14$$

      Then solve as normal:

$$Q_s = Q_d \Rightarrow 3p - 14 = 10 - 2p \Rightarrow 5p = 24 \Rightarrow \boldsymbol{p = \frac{24}{5}, Q = \frac{2}{5}}$$

   b. $Q_s = -20 + 3p$ and $Q_d = 220 - 5p$

      Supply curve shifts up by 3 units. New inverse supply curve is:

$$p = \frac{Q_s}{3} + \frac{29}{3} \Rightarrow Q_s = 3p - 29$$

      Can also be worked by substituting $p - 3$ into supply curve:

$$Q_s = -20 + 3(p - 3) = 3p - 29$$

      Then solve as normal:

$$Q_s = Q_d \Rightarrow 3p - 29 = 220 - 5p \Rightarrow 8p = 249 \Rightarrow \boldsymbol{p = \frac{249}{8}, Q = \frac{515}{8}}$$

   c. $Q_s = -45 + 8p$ and $Q_d = 125 - 2p$

      Supply curve shifts up by 3 units. New inverse supply curve is:

$$p = \frac{Q_s}{8} + \frac{69}{8} \Rightarrow Q_s = 8p - 69$$

      Can also be worked by substituting $p - 3$ into supply curve:

$$Q_s = -45 + 8(p - 3) = 8p - 69$$

      Then solve as normal:

$$Q_s = Q_d \Rightarrow 8p - 69 = 125 - 2p \Rightarrow 10p = 194 \Rightarrow \boldsymbol{p = \frac{194}{10}, Q = \frac{431}{5}}$$

7. (***Challenge***) Suppose that $Q_d = 34 - p$ and $p = 1 + Q_s$. Find the equilibrium price and quantity. Suppose that a specific tax of £1 per unit is levied on producers.

$$Q_s = Q_d \Rightarrow 34 - p = p - 1 \Longrightarrow 2p = 35 \Longrightarrow \boldsymbol{p} = \frac{\mathbf{35}}{\mathbf{2}}, \boldsymbol{Q} = \frac{\mathbf{33}}{\mathbf{2}}$$

a. What will be the new equilibrium price and quantity?

New supply curve:

$$p = 2 + Q^s$$

Finding equilibrium:

$$34 - p = p - 2 \Longrightarrow 2p = 36 \Longrightarrow \boldsymbol{p} = \mathbf{18}, \boldsymbol{Q} = \mathbf{16}$$

b. How much tax revenue will be collected by the government in this new equilibrium?

Recall that this is a per unit tax. Hence tax revenue is simply the tax multiplied by the equilibrium output: $1 \times 16 = \pounds\mathbf{16}$

c. Let's define the consumer burden of the tax as $P_{new} - P_{old}$, and the producer burden of the tax as $\tau - (P_{new} - P_{old})$. Who has the bigger tax burden?

$$CB: P_{new} - P_{old} = 18 - \frac{35}{2} = \mathbf{0.5}$$
$$PB: \tau - (P_{new} - P_{old}) = 1 - 0.5 = \mathbf{0.5}$$

The tax burden is the same for consumers and producers and is shared equally.

d. Suppose that the quantity demanded equation changes to $Q_d = 17 - \frac{1}{2}p$. Work out original equilibrium, the post-tax equilibrium, the tax revenue, and the consumer and producer tax burdens.

$$Q_s = Q_d \Rightarrow 17 - \frac{1}{2}p = p - 1 \Longrightarrow \frac{3}{2}p = 18 \Longrightarrow \boldsymbol{p} = \mathbf{12}, \boldsymbol{Q} = \mathbf{11}$$

New supply curve:

$$p = 2 + Q^s$$

Finding equilibrium:

$$17 - \frac{1}{2}p = p - 2 \Longrightarrow \frac{3}{2}p = 19 \Longrightarrow \boldsymbol{p} = \frac{\mathbf{38}}{\mathbf{3}}, \boldsymbol{Q} = \frac{\mathbf{32}}{\mathbf{3}}$$

Calculating tax burdens:

$$CB: P_{new} - P_{old} = \frac{38}{3} - 12 = \frac{\mathbf{2}}{\mathbf{3}}$$
$$PB: \tau - (P_{new} - P_{old}) = 1 - \frac{2}{3} = \frac{\mathbf{1}}{\mathbf{3}}$$

e. Comment on how the tax burdens have changed. Why do you think this has happened?

   The tax burden for the firm has decreased and the burden for the consumer has grown. This is because the elasticity of demand has decreased. This means that the responsiveness of consumer demand to changes in price are smaller. Hence, a firm is able to burden the consumer with an increase an price without losing much revenue.